Undersampling Pattern for Compressive Sampling MRI

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Abstract: In k-space, low frequency components are distributed around the center and high frequency components are at the periphery. These low and high frequency values represent the higher and lower energy samples respectively. The contrast of the MR image is mainly due to the higher energy samples. Hence more number of low frequency components is acquired for proper image reconstruction. Large amount of samples are collected near the middle region than the outer boundary surface. This process of collecting few data from which an image can be efficiently reconstructed is identified as Compressive Sensing or Compressive Sampling (CS). To get the undersampled data or k-space data sampling trajectories are applied on the fully sampled k-space. These sampling trajectories are generated using Probability Density Function (PDF). It is observed that proper image reconstruction is possible with only 40% of k-space data.

Keywords: Compressive Sampling, k-space, Undersampling, Probability Density Function, Sampling Trajectory.

Introduction

Magnetic Resonance Imaging (MRI) is a method that produces superior image quality of human body. This technique mainly works on the principle of Nuclear Magnetic Resonance (NMR). MRI uses neither radioactive materials nor high penetrating power. Hence this process will not be a reason to any hazard to patient's health.

In traditional sampling methods the images are sampled at Nyquist rate (sampling rate) which is double the largest frequency there in the signal of interest [1]. But in many applications resulting sampling rate is very high such that large number of samples needs to be considered [2]. This difficulty is solved by a method called compressive sampling. Using CS it is possible to recover images from small amount of measurements than using traditional techniques [3]. This strategy is used in MRI to minimize the imaging time by reconstructing the original image from lesser number of undersampled data.

In MRI k-space, higher energy is situated in the central region than the outer surface. Since energy allocation in the k-space is non uniform, a random undersampling technique will results in low frequency aliasing. The effect of such aliasing can be eliminated by taking more samples or finding an alternate method to make aliasing artifact incoherent. Therefore variable density random undersampling is used [4]. The optimum sampling trajectory is selected based on reconstruction performance. Such optimum sampling pattern should consider the energy distribution of k-space. Most widely used sampling pattern is straight lines from the Cartesian grid which is realistic when put into practice [5].

The objective of this work is to design an optimum sampling pattern to sample the k-space with the help of appropriate PDF that reduces MRI scan time significantly by using lesser sampling percentage. Thereby it produces a sampling trajectory which collects minimum number of samples from k-space. Based on the reconstruction performance this helps to recognize the excellent sampling pattern to sample the k-space.

Methodology

k-Space Generation

To generate the k-space using Fourier Transform, compute the Fourier Transform of the input MR image to transform it into Fourier domain. In the Fourier domain high frequency values are concentrated around the center and low frequency values are present at the corner [6]. So shift all low frequency values to the centre of the image which gives the k-space as shown in Fig. 1.

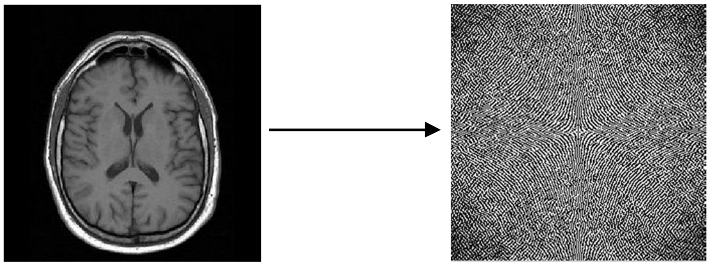

Figure1. Fourier Transform of the input MR image

Designing the PDF

The main objective of this work is to design a practical sampling scheme for conventional Cartesian MRI which minimises the scan time, thereby speeding up the process. This sampling method uses a variable-density sampling scheme with lesser number of samples close to the center of the k-space. The probability function for generating the Cartesian sampling pattern is given by,

$$p(\vec{r}) = \begin{cases} 1 & r \geq R \\ (1-r)^p & r < R \end{cases} \tag{1}$$

where R is a real number and $0 < R < 1$. The algorithm for the generation of Cartesian sampling pattern can be specified as,

1. Initialise: S_x, S_y, $pctg$, radius, p
2. $PC = S_x * S_y * pctg$ (2)
3. A matrix [x,y] is generated using linearly spaced vectors.
4. Compute r which is set of largest elements in a matrix [x,y]
5. $pdf = (1 - r^p)$ (3)
6. if (sum(pdf) > PC)
   idx = find (r < radius)
   pdf (idx) = 1
   end if

In this algorithm p is the power of polynomial which controls the number of samples to be taken and value of '$pctg$' controls the sampling percentage and both can be selected randomly by performing number of trials.

Creating the sampling trajectory

The sampling trajectory for Cartesian sampling scheme is designed using PDF. Using the generated PDF we can create the sampling mask according to energy distribution of k-space [7]. This sampling pattern is a variable density random matrix with elements as 0's and 1's, which samples the k-space only where its value is 1. Hence only the significant information from the k-space is acquired using CS. Thereby it gives an assurance that it is possible to perform the exact reconstruction of the original image using incomplete measurements [8]. The k-space also known as spatial frequency data is obtained by performing Fourier transform of the input MR image. The number of k-space lines that are need to be sampled from the fully acquired k-space is known as scan percentage which can be defined as,

$$Scan\ Percentage = \frac{Number\ of\ k\text{-}space\ lines\ acquired\ for\ undersampling}{Number\ of\ k\text{-}space\ lines\ for\ normal\ reconstruction} * 100 \tag{4}$$

Image Reconstruction

Image reconstruction is performed is using Discrete Fourier Transform (DFT) technique. The algorithm is given by,

1. Reading the k-space samples
2. Inverse Fast Fourier Transform (IFFT) in k_x direction
3. IFFT in k_y direction
4. IFFT shift
5. Image display

To define the better quality of image, reconstruction results are expressed in Structural Similarity Index Measure (SSIM). In this a comparison is done among the reconstructed and original image in terms of luminance, contrast and structure [9]. Other image quality measurements used are Mean Square Error (MSE) and Peak signal to Noise Ratio (PSNR). But their drawback is that all the images with same MSE do not indicate that they will have same distortion. Hence with PSNR and MSE the distortions in the images are not clearly visible. In order to overcome this drawback SSIM is used. Consider the luminance comparison function $l(a,b)$, contrast comparison function $c(a,b)$ and the structural comparison function $s(a,b)$ can be related to define SSIM between images a and b as,

$$SSIM(a,b) = l(a,b)c(a,b)s(a,b) \tag{5}$$

SSIM analyses the distortion of image as a combination of three factors. They are distortion in contrast and luminance and loss of correlation.

Results and Discussion

The simulation results are provided to establish the efficiency of the proposed work. The designed variable density sampling trajectory is applied to the input MR image. This algorithm is implemented in MATLAB 2013a and the input image to be sampled is of size 256x256 and shown in Fig. 2(a). The generated sampling patterns are variable density horizontal lines and vertical lines in Cartesian scheme which is shown in Fig. 2(b) and Fig. 2(c) respectively.

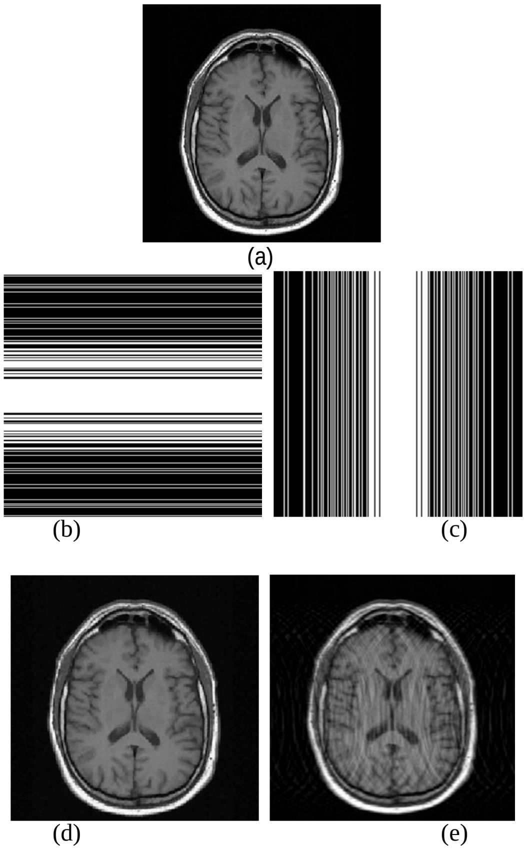

Figure2. Sampling in Cartesian scheme (a) Original MR image (b) Horizontal lines mask (40%) (c) Vertical lines mask (40%) (d)-(e) Reconstructed results using horizontal and vertical lines mask respectively

To compare the effectiveness of proposed method we need to reconstruct the MRI image from the samples obtained using horizontal and vertical lines Cartesian pattern and it is shown in Fig. 2(d) and Fig. 2(e) respectively. Table 1 summarizes the simulation results for the MRI image with two sampling trajectories where the power of polynomial ranges from 5 to 3. The eminence of reconstructed image is measured using SSIM.

In Fig. 2 it is observed that 40% horizontal lines mask and 40% vertical lines mask shows difference in their contrast in the reconstructed results. Using the variable density vertical lines mask it is possible to sample 40% of the input image. Compared to horizontal lines mask it shows lot of ringing artifacts in the resulting image. Vertical lines in k-space indicate the phase-encode axis and horizontal lines represents frequency-encode axis. Aliasing is commonly observed in vertical lines mask because phase encoding step takes complete MR excitation and thus it requires lot of time. To minimize the time we need to skip the phase encoding steps. Then the resulting image will be affected by aliasing. Therefore horizontal lines mask is preferred over vertical lines mask. Only in rare cases vertical lines mask shows improved results than horizontal lines mask.

Table 1.Quality measurements of reconstruction results. Bold font indicates the best performance with respect to SSIM

Sampling Pattern	p	SSIM
Horizontal lines mask	5	0.8006
	4	0.8887
	3	**0.9797**
Vertical lines mask	5	0.7667
	4	0.8320
	3	0.8466

The quality analysis of the simulation output for the reconstructed image is measured in SSIM. This shows difference between reconstructed and original image which is not noticeable with visual analysis.

Conclusions

In traditional sampling methods images are sampled at Nyquist rate in which large amount of data are collected during acquisition process [10]. Using CS only the significant information from the image can be gathered. Different sampling patterns are generated for numerous scan percentages and a variety of PDF parameters. The best sampling pattern is obtained for some values of PDF parameter and scan percentage. The horizontal lines mask which collects only 40% of k-space data is concluded as the best possible sampling trajectory based on the reconstruction performance.

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